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# INVENTORY CONTROL UNDER GAMMA DEMAND AND RANDOM LEAD TIME

by

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In a paper recently published in this journal, Keaton[1] showed how to include the gamma approximation of demand in a procedure proposed by Tyworth[2] to estimate reorder points in a random lead-time setting. Keaton observed that the gamma distribution is an especially effective choice, because it has such non-negative values as true demand and is flexible. These characteristics make the gamma distribution a reasonable model of demand for A, B, or C inventory items. In addition, the flexibility of the gamma distribution, coupled with the freedom to use any discrete distribution of lead time in Tyworth's procedure, means that the inventory planner can accurately estimate reorder points under a broad range of stochastic conditions. Keaton observed, nevertheless, that an important technical problem with the gamma distribution has been the lack of a simple way to calculate stockout probabilities and expected shortages. Such calculations, moreover, require a computer environment for convenient application. Thus Keaton contributed a PASCAL program that solves this problem and executes Tyworth's procedure. Using this program, an analyst can accurately estimate the reorder point ($s^*$) that satisfies either a stockout or a fill-rate service target under conditions of gamma demand and stochastic lead-time.

When using a continuous review system, however, the inventory planner is ultimately interested finding the reorder quantity ($Q$) and the reorder point ($s$) that, together, minimize total system costs for some pre-specified service, or shortage-cost, criterion. This will be a technically difficult task when either an order-fill or a unit shortage-cost criterion controls the ($s$, $Q$) inventory system. This difficulty arises because $s$ responds to changes in $Q$ in such inventory systems. Thus, although Keaton's PASCAL program faciliates the determination of reorder points, some nontrivial additional programming will be needed to find the optimal joint ($s$, $Q$) solutions conveniently. For example, one might first use the PASCAL program

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to develop a vector of the reorder points ($s^+$) that achieve the desired order-fill target over an appropriate range of $Q$ values. A solution could then be found in two ways. The first way is to create a table "lookup" logic to find $s^+$ for any given $Q$ in the array of $Q$ and $s^+$ values and then develop a procedure to search for the order quantity ($Q^*$) that, together with its directly corresponding reorder point ($s^+$), will minimize total costs. The second way is to fit a curve to the array of $Q$ and $s^+$ values and then determine an analytical expression for the economic order quantity.[3] Both methods are awkward, though, because they require a recalculation of the $Q$, $s^+$ array whenever the analyst wants to evaluate different order-fill (or unit-shortage-cost) levels or different demand and lead-time parameters.

Our purpose is to present a nonlinear programming approach for analyzing ($s$, $Q$) inventory systems in a gamma-demand and random lead-time setting. The approach follows the framework for setting reorder points that Tyworth proposed and is easily implemented in today's leading computer worksheets.

## APPROACH

We make all of the standard assumptions for the continuous review ($s$, $Q$) inventory system involving random demands and lead times. In addition, we consider two basic inventory control models as follows.

### Order-Fill ($P_2$) Service Model

Let $P_2$ represent a pre-specified fraction of annual sales order not filled and backordered. Given this policy, the expected total annual inventory-system cost *ETAC* is the sum of ordering ($c_o$) and holding ($c_h$) costs. For reorder quantity $Q$, annual unit volume $R$, holding cost factor $h$, unit value $v$, ordering cost $A$, and lead-time demand $X$ with mean $\mu_X$, we can express the cost elements as follows:

$$c_o = AR/Q \tag{1}$$

and

$$c_h = (Q/2 + s\text{-}\mu_X)vh \tag{2}$$

where $Q/2$ and $s\text{-}\mu_X$ represent the cycle and safety stocks, respectively. The model, therefore, can be expressed as follows

$$ETAC(s,Q) = AR/Q + (Q/2 + s\text{-}\mu_X)vh \tag{3}$$

The problem is to determine the values of $s$ and $Q$ that minimize *ETAC* and achieve the desired service ($P_2$) target. To solve this problem, we first translate the order fill policy into the target (planned) shortages and then develop a comparable measure for expected shortages.

Let *TS* define the target number of units short per replenishment cycle. For an order-fill target of $P_2$ with complete backordering, the target units short per year are $(1\text{-}P_2)R$. Since there are $R/Q$ cycles per year, we can define *TS* as $(1\text{-}P_2)R \div R/Q$ or simply as $(1\text{-}P_2)Q$.

Let *ES* represent the expected number of units short per replenishment cycle. For the case of gamma demand and stochastic lead time, we need to evaluate the gamma loss function to determine *ES*. We can define the loss function of the gamma distribution $G(s)$ as follows:

$$G(s) = \int_s^\infty (x_o - s) f_x(x_o) dx_o \tag{4}$$

where $s$ is the reorder point and $f_x(\cdot)$ is the gamma probability density function (pdf) of $X$. If $X$ represents the convolution of $D$ and $T$ and is assumed to have a gamma distribution, we can use the $G(s)$ to determine *ES* directly.

Tyworth's procedure, however, evaluates *ES* by treating $X$ as a set of conditional probability distributions of demand over lead time, rather than as the convolution of $D$ and $T$. Thus it is necessary to develop an expression for the conditional expected shortages per replenishment cycle, $E(s\ T = t)$. We accomplish this task as follows. First, let period demand $D$ have a gamma distribution denoted as $G(\alpha\beta)$, where $\alpha$ and $\beta$ are the shape and scale parameters. We can define the mean ($\mu_D$) and variance ($\sigma_D^2$) of D as $\alpha\beta$ and $\alpha\beta^{2,}$ respectively.[4] Conversely, $\alpha$ equals $\mu_D^2/\sigma_D^2$ $\beta$ equals $\mu_D^2/\sigma_D^2$. Further, the probability density function (pdf) of $D$ may be expressed as

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$$f(d \mid \alpha,\beta)=\frac{d^{\alpha-1}e^{-d/\beta}}{\beta^{\alpha}\Gamma(\alpha)} \tag{5}$$

where $\Gamma(\alpha)$ is the *complete gamma function* defined as

$$\Gamma(\alpha)=\int_0^{\infty} t^{\alpha-1}e^{-t}dt \tag{6}$$

Second, let lead time $T$ assume any discrete form with probability mass function (pmf) $P_t = P(T=t)$. The conditional distribution of demand given $T=t$, for $t=1,2,\ldots,m$ is $G(t\alpha,\beta)$, where $m$ is the maximum lead-time. We now can define the conditional expected units short for $T=t$ as

$$ES_t=E(s \mid T=t)=\int_s^{\infty}(u-s)dG(t\alpha,\beta) \tag{7}$$

After some algebra, we obtain

$$E(s \mid T=t) = t\alpha\beta(1-G_1(s))-s(1-G_0(s)), \tag{8}$$

where $G_0$ and $G_1$ are the cumulative distributions of gamma functions $G(t\alpha,\beta)$ and $G(t\alpha+1,\beta)$, respectively. The expected units short per replenishment cycle ($ES$) is then defined as the weighted average of the conditional expected shortages:

$$ES = \sum_{t=1}^{m} P_t ES_t \tag{9}$$

An analyst may obtain the discrete lead time probability loadings $P_t$ by using company data to develop a simple histogram of lead times. Alternatively, one may model lead time as a common statistical form—say, gamma, Weibull, or exponential—and then develop discrete estimates of $P_t$ for that particular form. This task is easily accomplished as follows. Let $F(y)$ be the cumulative distribution function of lead time. We can obtain the probability $P_t$ from the following formula

$$P_t = F(t) - F(t-1) \tag{10}$$

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We can now reformulate the problem as follows: determine $s,Q$ to minimize $ETAC$ subject to the constraints that

$$ES \leq TS$$

$$s \geq 0$$

$$Q \geq 1$$

As demonstrated in the next section of this paper, this problem can be solved easily with well established nonlinear programming methods.

### Unit Shortage Cost ($B_2$) Model

The unit shortage cost model includes a pre-specified fractional charge per unit short ($B_2$), which is an estimate of the cost of a stockout including the effects of disservice on future sales. The expected total annual cost expression for this model is:

$$ETAC(s,Q) = AR/Q + (Q/2 + s - \mu_X)vh + ES \bullet B_2 vR/Q \tag{11}$$

The problem now is to determine $s$ and $Q$ to minimize $ETAC$ subject to the constraints that $s$ is non-negative and $Q$ is at least one unit. Again, nonlinear programming methods can be used to solve this problem.

## IMPLEMENTING THE MODELS

We provide three numerical examples to illustrate practical methods of finding solutions to the problems previously discussed. The first example replicates the illustration used by Keaton to demonstrate his PASCAL program.[5] The next two examples demonstrate how to determine optimal ($s^*$, $Q^*$) solution for the fill-rate ($P_2$) model and the shortage cost ($B_2$) model.

Table 1 presents an illustration of Keaton's numerical example in Microsoft Excel 5.0. The period demand $D$ is shown as gamma (2, .5) in the Parameters Section of the model, rather than as gamma (2,2) in Keaton's example, because Keaton's $\beta$ is equivalent to $1/\beta$ in our formulation of the gamma distribution and its parameters. The Problem/Solution Section of the worksheet identifies the single

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## TABLE 1

## KEATON'S NUMERICAL EXAMPLE

| | C | D | E | F | G | H |
|---|---|---|---|---|---|---|
| 1 | **GIVEN PARAMETERS** | | | | | |
| 2 | | | | | | |
| 3 | **Element** | **Name** | **Value** | **Lead Time (T)** | | |
| 4 | ***Gamma Demand*** | | | t | p(t) | |
| 5 | α | ALPHA | 2.00 | 1 | 0.350 | |
| 6 | β | BETA | 0.50 | 2 | 0.500 | |
| 7 | Order Size | Q | 20 | 3 | 0.150 | |
| 8 | Fill-rate $P_2$ | P | 0.98 | | | |

| | J | K | L |
|---|---|---|---|
| 1 | **PROBLEM FORMULATION AND SOLUTION** | | |
| 3 | Element | Name | Value |
| 4 | ***Problem*** | | |
| 5 | ***Decision Variable*** | | |
| 6 | Reorder level | s | 1.945 |
| 7 | ***Objective Function*** | | |
| 8 | Set ES = TS | | |
| 9 | ***Constraints*** | | |
| 10 | Q ≥ 1, s ≥ 0 | | |
| 11 | ***Solution*** | | |
| 12 | ***Shortages*** | | |
| 13 | Expected short/cycle | ES | 0.400 |
| 14 | Target short/cycle | TS | 0.400 |
| 15 | Actual order-fill level | PX | 98.00% |

**WORK AREA**

***Lead-Time Probability Loadings***

| T | 1 | 2 | 3 |
|---|---|---|---|
| p(t) | 0.350 | 0.500 | 0.150 |
| Sum[p(t)] | 1.000 | | |

***Gamma Loss Function***

| | | | |
|---|---|---|---|
| E(s\|t) | 0.06026 | 0.41537 | 1.14172 |
| p(t)*E(s\|t) | 0.02109 | 0.20769 | 0.17126 |

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decision variable $s$, the objective function $ES$, and the constraint $s \geq 0$. For a given positive value of $Q$, the goal is to find a non-negative value of $s$ so that $ES = TS$. The solution is found easily by using the math programming engine ("Solver" tool) in Excel. This tool is based on well established numerical methods for equation solving and optimization, including the best feasible gradient search algorithm and a conjugate gradient method. The Work Area Section contains a copy of the given lead-time probability loadings and the conditional expected shortages, $E(s \mid T = t)$. Excel's GAMMADIST(•) function makes it easy to use equation (8) to determine the expected conditional shortages over the entire range of lead times and then computer $ES$ from equation (9).

Table 2 illustrates an order-fill ($P_2$) service model that incorporates the parameters shown in Table 1, along with a unit price ($v$) of \$100, an ordering cost ($A$) of \$5, and a holding cost factor ($h$) of 30 percent. The Problem/Solution Section of this model has two decision variables ($s$, $Q$), a new objective function ($ETAC$), and a different set of constraints ($ES \leq TS$, $s \geq 0$ and $Q \geq 1$). Additionally, we used an integer constraint for $Q$ to make our units of measure consistent with Keaton's example in Table 1. The analyst again uses the Solver tool to minimize $ETAC$ by changing both $s$ and $Q$ subject to the given constraints. The optimal value of $ETAC$ is \$299.92, with $Q^* = 10$ and $s^* \sim 2.63$.

We use Table 3 to validate the results. Each pair of $Q$ and $s^+$ in this table satisfies the constraint that $ES = TS$ or, equivalently, the order-fill target. The directly corresponding calculations of $ETAC$ for each $Q$, $s^+$ pair, however, demonstrate that $ETAC(\bullet)$ is convex in $Q$ given $s^+$ (see Figure 1). These results confirm that $Q^* = 10$, $s^* \sim 2.63$, and $ETAC^* = \$299.92$.

Table 4 illustrates the shortage-cost ($B_2$) model. This time the task is to minimize $ETAC$ as defined in equation (11) by changing $s$ and $Q$, subject to the constraints that $Q \geq 1$ and $s \geq 0$. In our example, we once again imposed an integer constraint on $Q$ to obtain units of measure consistent with Keaton's example. The results show that, for $B_2 \sim 7$ percent, the solution is $Q^* = 10$, $s^* \sim 2.85$, and $ETAC^* = \$334.15$. Finally, Tables 5 and 6 document the worksheet formulas of the order-fill model and the unit-short cost models shown in Tables 2 and 4, respectively..

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## TABLE 2

## ORDER-FILL MODEL

### GIVEN PARAMETERS

| Element | Name | Value |
|---|---|---|
| *Gamma Demand* | | |
| α | ALPHA | 2.00 |
| β | BETA | 0.50 |
| Periods/yr | PY | 250 |
| | | |
| *Inventory* | | |
| Holding factor | H | 30.0% |
| Order fill target | P | 98.0% |
| Order cost | A | $5 |
| Unit value | V | $100 |

| Lead Time (T) | |
|---|---|
| t | p(t) |
| 1 | 0.350 |
| 2 | 0.500 |
| 3 | 0.150 |

### PROBLEM/SOLUTION

| Element | Name | Value |
|---|---|---|
| ***Problem*** | | |
| *Decision Variables* | | |
| Order size | Q | 10 |
| Reorder level | s | 2.631 |
| *Objective Function* | | |
| MIN: Expected total cost ETAC | | $299.92 |
| *Constraints* | | |
| ES ≤ TS | | |
| Q ≥ 1,s ≥ 0 | | |
| ***Solution*** | | |
| *Shortages* | | |
| Expected short/cycle | ES | 0.200 |
| Target short/cycle | TS | 0.200 |
| Actual order-fill level | PX | 98.00% |
| *Stock Levels* | | |
| Cycle | CS | 5.00 |
| Safety | SS | 0.83 |
| *Cost Analysis* | | |
| Cycle stock holding | CSC | $150.00 |
| Safety stock holding | SSC | $24.92 |
| Ordering/setup | OC | $125.00 |

### WORK AREA

| Element | Name | Value |
|---|---|---|
| *Demand* | | |
| Avg. period demand | UD | 1.00 |
| Std. dev. demand | SD | 0.71 |
| Annual demand | R_ | 250 |
| Avg. lead-time demand | UL | 1.80 |
| *Replenishment* | | |
| Lead time | UT | 1.80 |
| Orders/yr | RY | 25.00 |
| Cycle length | CL | 10.00 |

*Lead-Time Probability Loadings*

| T | 1 | 2 | 3 |
|---|---|---|---|
| p(t) | 0.350 | 0.500 | 0.150 |
| Sum [p(t)] | 1.000 | | |

*Gamma Loss Function*

| | | | |
|---|---|---|---|
| E(s\|t) | 0.019 | 0.186 | 0.669 |
| p(t)* E(s\|t) | 0.007 | 0.093 | 0.100 |

*Lead Time Computation Elements*

| | | | |
|---|---|---|---|
| Tp(t) | 0.350 | 1.000 | 0.450 |
| Sum (Tp(t)) | 1.800 | | |

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## TABLE 3

## $Q$, $s^+$ AND *ETAC*

| $Q$ | $s^+ \mid Q$ | $ETAC \mid Q$ |
|---|---|---|
| 1 | 4.589 | $1,348.67 |
| 2 | 4.035 | 722.05 |
| 3 | 3.698 | 518.62 |
| 4 | 3.454 | 422.12 |
| 5 | 3.261 | 368.82 |
| 6 | 3.100 | 337.32 |
| 7 | 2.960 | 318.37 |
| 8 | 2.839 | 307.42 |
| 9 | 2.729 | 301.77 |
| 10 | 2.631 | 299.92 |
| 11 | 2.540 | 300.84 |
| 12 | 2.457 | 303.87 |
| 13 | 2.379 | 308.52 |
| 14 | 2.306 | 314.48 |
| 15 | 2.238 | 321.47 |
| 16 | 2.174 | 329.33 |
| 17 | 2.112 | 337.88 |
| 18 | 2.054 | 347.06 |
| 19 | 1.998 | 356.73 |
| 20 | 1.945 | 366.84 |
| 21 | 1.894 | 377.33 |
| 22 | 1.844 | 388.15 |
| 23 | 1.797 | 399.26 |
| 24 | 1.751 | 410.62 |
| 25 | 1.707 | 422.21 |
| 26 | 1.664 | 433.99 |
| 27 | 1.622 | 445.96 |
| 28 | 1.582 | 458.09 |
| 29 | 1.542 | 470.38 |
| 30 | 1.504 | 482.79 |

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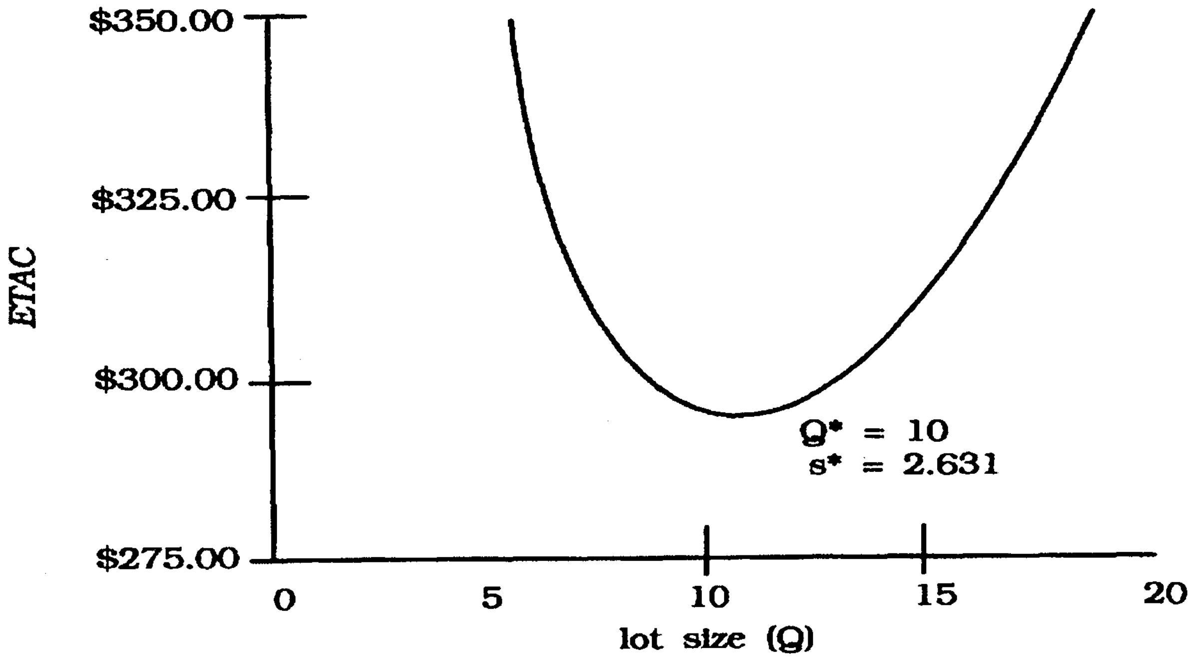
FIGURE 1

EXPECTED TOTAL ANNUAL COSTS (ETAC)
FOR ORDER-FILL MODEL



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## TABLE 4

## UNIT-SHORT COST MODEL

| C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

**GIVEN PARAMETERS**

| Element | Name | Value |
|---|---|---|
| *Gamma Demand* | | |
| α | ALPHA | 2.00 |
| β | BETA | 0.50 |
| Periods/yr | PY | 250 |
| | | |
| *Inventory* | | |
| Holding factor | H | 30.0% |
| Unit-short factor $B_2$ | B | 7.0% |
| Order cost | A | $5 |
| Unit value | V | $100 |

| Lead Time (T) | |
|---|---|
| t | p(t) |
| 1 | 0.350 |
| 2 | 0.500 |
| 3 | 0.150 |

**PROBLEM/SOLUTION**

| Element | Name | Value |
|---|---|---|
| ***Problem*** | | |
| *Decision Variables* | | |
| Order size | Q | 10 |
| Reorder level | s | 2.854 |
| *Objective Function* | | |
| MIN: Expected total cost ETAC | | $334.15 |
| *Constraints* | | |
| Q ≥ 1,s ≥ 0 | | |
| ***Solution*** | | |
| *Shortages* | | |
| Expected short/cycle | ES | 0.157 |
| Resulting order-fill | PX | 98.43% |
| *Stock Levels* | | |
| Cycle | CS | 5.00 |
| Safety | SS | 1.05 |
| *Cost Analysis* | | |
| Shortages | SC | $27.53 |
| Cycle stock holding | CSC | $150.00 |
| Safety stock holding | SSC | $31.62 |
| Ordering/setup | OC | $125.00 |

**WORK AREA**

| Element | Name | Value |
|---|---|---|
| *Demand* | | |
| Avg. period demand | UD | 1.00 |
| Std. dev. demand | SD | 0.71 |
| Annual demand | R_ | 250 |
| Avg. lead-time demand | UL | 1.80 |
| | | |
| *Replenishment* | | |
| Lead time | UT | 1.80 |
| Orders/yr | RY | 25.00 |
| Cycle length | CL | 10.00 |

***Lead-Time Probability Loadings***

| T | 1 | 2 | 3 |
|---|---|---|---|
| p(t) | 0.350 | 0.500 | 0.150 |
| Sum [p(t)] | 1.000 | | |

***Gamma Loss Function***

| | | | |
|---|---|---|---|
| E(s\|t) | 0.013 | 0.141 | 0.550 |
| p(t)* E(s\|t) | 0.004 | 0.070 | 0.083 |

***Lead Time Computation Elements***

| | | | |
|---|---|---|---|
| Tp(t) | 0.350 | 1.000 | 0.450 |
| Sum (Tp(t)) | 1.800 | | |

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## TABLE 5

## ORDER-FILL MODEL FORMULAS

| | K | L | M | N | O | P | Q | R | S | T | U |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | WORK AREA | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | Name | Value | | Element | Name | Value | | *Lead-Time* | | | |
| 4 | | | | ***Demand*** | | | | T | 1 | 2 | 3 |
| 5 | | | | Avg. period demand | UD | =ALPHA*BETA | | p(t) | =H5 | =H6 | =H7 |
| 6 | Q | 10 | | Std. dev. demand | SD | =SQRT(ALPHA*BETA^2) | | Sum[p(t)] | =SUM(S5:U5) | | |
| 7 | s | 2.630757 | | Annual demand | R_ | =UD*PY | | | | | |
| 8 | | | | Avg. lead-time demand | UL | =UD*UT | | *Gamma Loss Function* | | | |
| 9 | ETAC | SUM(1.22:1.24) | | | | | | E(s\|t) | S$4*ALPHA*BETA*(1-GAMMADIST(s.(S$4*ALPHA) +1.BETA,TRUE))-s*(1-GAMMADIST(s,(S$4*ALPHA),BETA TRUE)) | copy of S9 | copy of S9 |
| 10 | | | | | | | | p(t)*E(s\|t) | =S9*S$5 | =T9*&T$5 | =U9*U$5 |
| 11 | | | | ***Replenishment*** | | | | | | | |
| 12 | | | | Lead time | UT | =S14 | | *Lead-Time* | | | |
| 13 | | | | Orders/yr | RY | =R_/Q | | Tp(t) | =S$4*S$5 | =T$4*T$5 | =U$4*U$5 |
| 14 | | | | Cycle length | CL | =PY/RY | | Sum(Tp(t)) | =SUM($S$13:$U$13) | | |
| 15 | ES | =SUM(S10:U10) | | | | | | | | | |
| 16 | TS | =(1-P)*Q | | | | | | | | | |
| 17 | PX | =1-ES/Q | | | | | | | | | |
| 18 | | | | | | | | | | | |
| 19 | CS | =Q/2 | | | | | | | | | |
| 20 | SS | =MAX(s-UL,0) | | | | | | | | | |
| 21 | | | | | | | | | | | |
| 22 | CSC | =CS*V*H | | | | | | | | | |
| 23 | SSC | =SS*V*H | | | | | | | | | |
| 24 | OC | =R_/Q*A | | | | | | | | | |
| 25 | | | | | | | | | | | |

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## TABLE 6

## UNIT-SHORTAGE COST MODEL FORMULAS

|  | K | L | M | N | O | P | Q | V |
|---|---|---|---|---|---|---|---|---|
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 | Name | Value |  | Element | Name | Value |  |  |
| 4 |  |  |  | *Demand* |  |  |  |  |
| 5 |  |  |  | Avg. period demand | UD | =ALPHA*BETA |  |  |
| 6 | Q | 10 |  | Std. dev. demand | SD | =SQRT(ALPHA*BETA^2) |  |  |
| 7 | s | 2.85415907120845 |  | Annual demand | R_ | =UD*PY |  |  |
| 8 |  |  |  | Avg lead-time demand | UL | =UD*UT |  |  |
| 9 | ETAC | =SUM(L21:L24) |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |
| 11 |  |  |  | *Replenishment* |  |  |  |  |
| 12 |  |  |  | Lead Time | UT | =S14 |  |  |
| 13 |  |  |  | Orders/yr | RY | =R_/Q |  |  |
| 14 |  |  |  | Cycle length | CL | =PY/RY |  |  |
| 15 | ES | =SUM(S10:U10) |  |  |  |  |  |  |
| 16 | PX | =1-ES/Q |  |  |  |  |  |  |
| 17 |  |  |  |  |  |  |  |  |
| 18 | CS | =Q/2 |  |  |  |  |  |  |
| 19 | SS | =MAX(s-UL,0) |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |
| 21 | SC | =ES*B*V*R_/Q |  |  |  |  |  |  |
| 22 | CSC | =CS*V*H |  |  |  |  |  |  |
| 23 | SSC | =SS*V*H |  |  |  |  |  |  |
| 24 | OC | =R_/Q*A |  |  |  |  |  |  |

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## CONCLUSIONS

We have provided a nonlinear programming approach for solving continuous review ($s,Q$) inventory system problems under gamma-demand and random lead-time conditions. These problems include both service and shortage models. In addition, we have demonstrated how these kinds of models can be conveniently executed in a computer worksheet. Using the worksheet, an inventory planner can easily determine the reorder point and lot size that jointly optimize system costs.

## NOTES

[1] Mark Keaton, "Inventory Control Under Gamma Demand and Stochastic Lead Time," *Journal of Business Logistics* 16, no. 1 (1995): 107-131.

[2] John E. Tyworth, "Modeling Transportation-Inventory Tradeoffs in a Stochastic Setting," *Journal of Business Logistics* 13, no. 2 (1992): 97-124.

[3] J. Banks and W. J. Fabrycky, *Procurement and Inventory Analysis* (Englewood Cliffs, N.J.: Prentice-Hall, 1987), pp. 119-139.

[4] See, for example, A. M. Law and W. D. Kelton, *Simulation Modeling and Analysis*, 2nd ed. (New York: McGraw-Hill, 1991), pp. 331-333.

[5] Same reference as Note 1, p. 128.

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